

Fractional periodic persistent current in a twisted normal metal loop: an exact result

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Abstract

We explore a novel mechanism for control of periodicity in persistent current in a twisted normal metal loop threaded by a magnetic flux ϕ . A simple tight-binding model is used to describe the system. Quite interestingly we see that, depending on the number of twist p , the persistent current exhibits ϕ_0/p ($\phi_0 = ch/e$, the elementary flux-quantum) periodicity. Such fractional periodicity provides a new finding in the study of persistent current.

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1 Introduction

Over the last few decades, the physics at sub-micron length scale provides enormous evaluation both in terms of our understanding of basic physics as well as in terms of the development of revolutionary technologies. In this length scale, the so-called mesoscopic or nanoscopic regime, several characteristic quantum length scales for the electrons such as system size and phase coherence length or elastic mean free path and phase coherence length are comparable. Due to the dominance of the quantum effects in the mesoscopic/nanososcopic regime, intense research in this field has revolved its richness. The most significant issue is probably the persistent currents in small normal metal rings. In thermodynamic equilibrium, a small metallic ring threaded by magnetic flux ϕ supports a current that does not decay dissipatively even at non-zero temperature. It is the well-known phenomenon of persistent current in mesoscopic normal metal rings which is a purely quantum mechanical effect and gives an obvious demonstration of the Aharonov-Bohm effect.¹ An electron moving around the ring without entering the region of magnetic flux, feels no classical force during its motion. But the quantum state of this electron is affected by changing the phase of its wave function due to the presence of the magnetic vector potential \vec{A} , related to the magnetic field \vec{B} through the relation $\vec{B} = \vec{\nabla} \times \vec{A}$. Accordingly, both the thermodynamic and kinetic properties vary with the magnetic flux ϕ . The possibility of persistent current was predicted in the very early days of quantum mechanics by Hund,² but their experimental evidences came much later only after realization of the mesoscopic systems. In 1983, Büttiker *et al.*³ predicted theoretically that persistent current can exist in mesoscopic normal metal rings threaded by a magnetic flux ϕ , even in the presence of impurity. The first experimental verification of persistent current in mesoscopic normal metal ring has been established in the pioneering experiment of Levy *et al.*⁴, and later, the existence of the persistent current was further confirmed by several experiments.^{5–8} Though the phenomenon of persistent current has been addressed quite extensively over the last two decades both theoretically^{9–29} as well as experimentally,^{4–8}

but the controversy between the theory and experiment cannot be resolved yet. As illustrative example, the major controversies appear in the determinations of (I) the current amplitude, (II) flux-quantum periodicities, (III) low-field magnetic susceptibilities, etc. In recent works,^{23–25} it has been predicted that the higher order hopping integrals, in addition to the nearest-neighbor hopping (NNH) integral, have a significant role to enhance the current amplitude (even an order of magnitude). The inclusion of the higher order hopping integrals with the NNH integral is much more convenient for the description of a system rather than the traditional NNH model, since the overlap of the atomic orbitals between various neighboring sites are usually non-vanishing. In other recent work,²⁶ we have focused that the low-field magnetic susceptibility can be mentioned exactly only for the one-channel systems with fixed number of electrons, while for all other cases it becomes random. To grasp the experimental behavior of the persistent current, one has to focus attention on the interplay of quantum phase coherence, disorder and electron-electron correlation and this is a highly challenging problem.

In this present paper, we concentrate ourselves on a strange behavior of persistent current which is quite different than that of the above mentioned issues. This is the appearance of the fractional periodic persistent current in a twisted normal metal loop, which is solely different from the conventional systems i.e., one-channel rings or multi-channel cylinders. Several interesting phenomena of persistent currents have already been studied in some particular twisted systems. For example, Yakubo *et al.*³⁰ have described the behavior of persistent currents in a twisted geometry, so-called the Möbius strip, and found the distinct flux-periodicity than that of an ordinary cylindrical sample. In other work, Ferreira *et al.*³¹ have also investigated the topological effect on persistent currents in a Möbius strip and revealed some interesting new results. Motivated with such kind of systems, in this paper we focus our study on the behavior of persistent current in a p -fold twisted loop geometry (see Fig. 1). Quite interestingly from our study we see that, depending on the number of twist p , the persistent current exhibits ϕ_0/p flux-quantum periodicity.

This phenomenon is completely opposite compared to any traditional ring/cylindrical system, where the current exhibits only ϕ_0 flux-quantum periodicity.

We organize this paper specifically as follows. In Section 2, we describe the model and the theoretical description for our calculations. Section 3 focuses the significant results and the discussion. Here we compute the persistent currents both for the systems with fixed number of electrons and fixed chemical potential and focus on the basic mechanism for the control of periodicity of persistent currents in a twisted geometry. Finally, we draw our conclusions in Section 4.

2 The model and the method

The schematic representation of a twisted normal metal loop threaded by a magnetic flux ϕ is shown

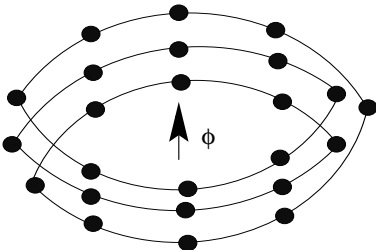


Figure 1: Schematic view of a twisted normal metal loop threaded by a magnetic flux ϕ . The filled circles correspond to the position of the atomic sites.

in Fig. 1. The system can be described by a single-band tight-binding Hamiltonian and within the non-interacting picture, the Hamiltonian looks in this form

$$H = \sum_{i=1}^{pN} \epsilon_i c_i^\dagger c_i + v \sum_{\langle ij \rangle} \left[e^{i\theta} c_i^\dagger c_j + e^{-i\theta} c_j^\dagger c_i \right] \quad (1)$$

In this expression, p is the total number of turns and N corresponds to the total number of atomic sites in each turn of the loop, ϵ_i 's are the site energies, c_i^\dagger (c_i) is the creation (annihilation) operator of an electron at site i of the loop, and v is the hopping

integral between nearest-neighbor sites in each turn. Here, $\theta = 2\pi\phi/N$ is the phase factor due to the flux ϕ (measured in units of $\phi_0 = ch/e$, the elementary flux quantum). Now in order to introduce the impurities in the system, we choose the site energies (ϵ_i 's) from an incommensurate potential distribution function: $\epsilon_i = W \cos(i\lambda\pi)$, where W is the strength of the potential and λ is an irrational number, and as a typical example we take it as the golden mean $(1 + \sqrt{5})/2$. Setting $\lambda = 0$, we get back the pure system with identical site potential W . The idea of considering such an incommensurate potential rather than the usual random distribution is that, for such a correlated disorder we do not require any configuration averaging and therefore the numerical calculations can be done in the low cost of time.

At absolute zero temperature, the persistent currents in the loop threaded by a flux ϕ is determined by¹⁰

$$I(\phi) = -\frac{\partial E(\phi)}{\partial \phi} \quad (2)$$

where, $E(\phi)$ is the ground state energy. We evaluate this energy exactly to understand unambiguously the anomalous behavior of persistent current, and this is achieved by exact diagonalization of the tight-binding Hamiltonian Eq. (1). Throughout the calculations, we take $v = -1$ and use the units where $c = 1$, $e = 1$ and $\hbar = 1$.

3 Results and discussion

In this section, we investigate the behavior of persistent currents both for the ordered and disordered twisted loops and illustrate how the periodicity of the current can be controlled by tuning the total number of twist in such a particular geometry. To have a deeper insight to the problem, let us first start our discussion with the energy-flux characteristics of a twisted loop. In an ordered system, we put $\epsilon_i = 0$ for all i in the above Hamiltonian given by Eq. (1), and the energy of the n -th single-particle state can be calculated analytically which is written as

$$E_n(\phi) = 2v \cos \left[\frac{2\pi}{pN} (n + p\phi) \right] \quad (3)$$

where n is an integer and restricted in the range: $-pN/2 \leq n < pN/2$. For the disordered case, since the energy E_n cannot be done analytically, we evaluate it by exact numerical diagonalization of the tight-binding Hamiltonian (Eq. (1)). As illustrative example, in Fig. 2 we present the energy levels of 4-fold ($p = 4$) twisted loops, where we fix the total number

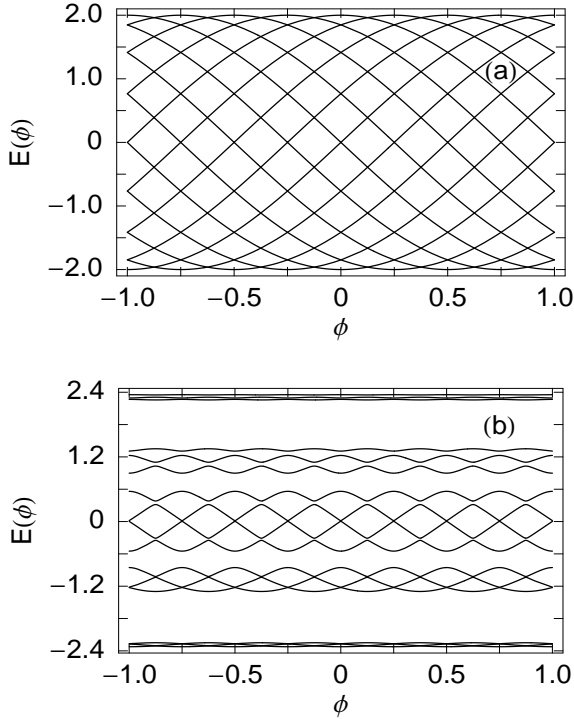


Figure 2: $E(\phi)$ - ϕ curves for (a) ordered ($W = 0$) and (b) disordered ($W = 1$) 4-fold ($p = 4$) twisted loops with total number of atomic sites in each turn $N = 4$.

of atomic sites $N = 4$ in each turn. Figures 2(a) and (b) correspond to the ordered and dirty systems respectively. In the absence of any impurity i.e., for perfect loop, the energy levels intersect with each other at $\phi = 0$ or at the integer multiple of ϕ_0/p . This indicates that the energy levels have degeneracy at these respective values of the magnetic flux ϕ . On the other hand, all these degeneracies move out as long as the impurities are given in the system, and

gaps open at the points of intersection. Accordingly, the energy levels vary continuously with respect to the flux ϕ (see the curves in Fig. 2(b)). Both these intersecting behavior and the continuous variation of the energy levels with ϕ are quite similar in nature as observed in traditional one-channel rings or multi-channel cylinders.²⁶ But the significant feature that is observed from the energy spectra is that, the energy levels show $\phi_0/4$ flux-quantum periodicity, instead of ϕ_0 . This ϕ_0 periodicity is observed only for the untwisted loops. One can also get the energy levels with other fractional periodicity by tuning the total number of twist p in such a twisted loop. This phenomenon is really very interesting, and since the energy levels are ϕ_0/p periodic in ϕ , the persistent current will also exhibit the ϕ_0/p oscillating behavior, which we will describe now. In the forthcoming sub-sections, we present some analytical as well as numerical calculations and study the behavior of persistent currents I in twisted geometries as a function of the flux ϕ , system size L , total number of electrons N_e , chemical potential μ , total number of twist p and the strength of disorder W .

3.1 System with fixed number of electrons N_e

In the absence of any impurity, the current carried by the n -th energy eigenstate is

$$I_n(\phi) = \left(\frac{4\pi v}{N} \right) \sin \left[\frac{2\pi}{pN} (n + p\phi) \right] \quad (4)$$

At absolute zero temperature, we can write the total persistent current in the following form

$$I(\phi) = \sum_n I_n(\phi) \quad (5)$$

For the systems with constant N_e , the total current will be obtained by taking the sum of the individual contributions from the lowest N_e energy levels. The variation of the persistent currents with flux ϕ for some typical twisted loops is represented in Fig. 3. Figures 3(a) and (b) correspond to the results for the 4-fold and 5-fold twisted loops respectively, where the total number of atomic sites in both the two cases

is taken as 100. The currents are determined for the fixed number of electrons $N_e = 40$, where the solid and dotted curves represent the currents for the perfect ($W = 0$) and disordered ($W = 1$) systems respectively. From the results it is observed that, in the ordered loops the current exhibits the

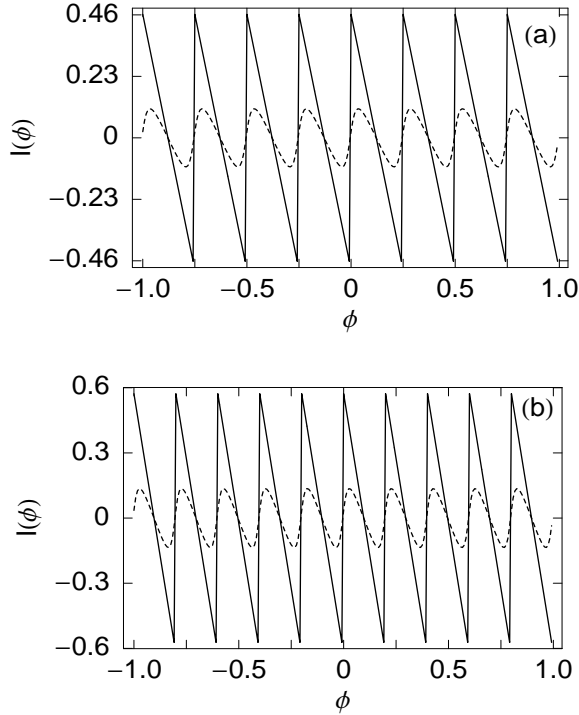


Figure 3: $I(\phi)$ - ϕ curves for (a) 4-fold ($p = 4$) and (b) 5-fold ($p = 5$) twisted loops with fixed number of electrons $N_e = 40$. The total number of atomic sites for both the two cases is 100, where we set $N = 25$ in (a) and $N = 20$ in (b). The solid and dotted curves correspond to the perfect ($W = 0$) and disordered ($W = 1$) systems respectively.

saw-tooth like nature, while it shows the continuous variation with much reduced amplitude in the disordered cases. These sharp transitions in the persistent currents for the ordered systems at the different values of ϕ appear due to the degeneracy of the energy eigenstates at these respective points. On the

other hand, for the disordered systems since there is no degeneracy in the energy levels, the current varies continuously with the flux ϕ . Now for these disordered systems, we get much reduced current amplitude compared to the perfect cases and this is due to the localization effect of the energy eigenstates.³² Traditional wisdom is that, the larger the disorder stronger the localization³² and accordingly, we will get lesser and lesser current amplitude with the increase of the disorder strength W . These phenomena are well established in the literature. But the most remarkable feature that is observed from these current-flux characteristics is that, the periodicity of the current changes as we change the total number of twist p . Our results show that for a fixed system size (here the total number of atomic sites is 100 for both the two cases), the periodicity of the current changes from $\phi_0/4$ (see Fig. 3(a)) to $\phi_0/5$ (see Fig. 3(b)) as we change the number of twist from $p = 4$ to $p = 5$. This phenomenon can be explained as follows. For a p -fold twisted loop, once an electron starts its motion along the loop from a point, it comes back to its original position after traversing the p -th turn and accordingly, it encloses $p\phi$ flux. This originates the ϕ_0/p oscillations in the persistent currents, instead of ϕ_0 oscillations as observed in conventional untwisted loops.²⁶ Thus by tuning the total number of twist, we can control the periodicity of persistent current. This really provides an interesting finding in this particular study.

3.2 System with fixed chemical potential μ

Now we discuss the characteristic features of the persistent currents for some twisted loops those are described with fixed chemical potential μ , instead of the fixed number of electrons N_e . For certain choices of μ , the system will have a fixed number of electrons, while for all other cases the number of electrons will vary as a function of ϕ . At absolute zero temperature, the total current for such systems (systems with fixed μ) will be obtained by adding the individual contributions from the energy levels with energies less than or equal to μ . As illustrative example, in Fig. 4 we plot the current-flux characteristics for some 3-fold

and 6-fold twisted loops respectively, where the total number of atomic sites in both these two cases is taken as 180. All the currents are computed for the fixed chemical potential $\mu = -1.0$, where the solid and dotted curves represent the same meaning as in Fig. 3. Similar to the previous systems, here we also

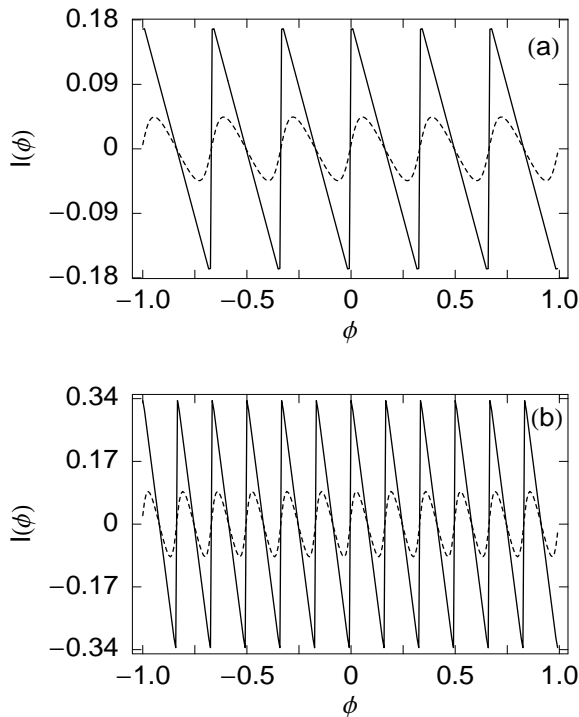


Figure 4: $I(\phi)$ - ϕ curves for (a) 3-fold ($p = 3$) and (b) 6-fold ($p = 6$) twisted loops with fixed chemical potential $\mu = -1.0$. The total number of atomic sites for both the two cases is 180, where we set $N = 60$ in (a) and $N = 30$ in (b). The solid and dotted curves correspond to the perfect ($W = 0$) and disordered ($W = 1$) systems respectively.

get the saw-tooth like nature in the persistent currents for the perfect loops, while they vary continuously with much reduced amplitude in the presence of impurity. Our results clearly show that for a fixed system size (here the total number of atomic sites is 180 for both the two cases), the current changes

its periodicity from $\phi_0/3$ (see Fig. 4(a)) to $\phi_0/6$ (see Fig. 4(b)) as we tune the number of twist from $p = 3$ to $p = 6$. The explanation of this behavior is exactly similar to that as mentioned earlier. Thus we can emphasize that, the periodicity of the persistent currents can be controlled by tuning the total number of twist in these geometries, whether they are described with fixed N_e or μ .

4 Concluding remarks

In conclusion, we have established a novel feature for control of the periodicity of persistent current in a twisted normal metal loop threaded by a magnetic flux ϕ . We have used a simple tight-binding model to describe the system and provided some analytical as well as numerical calculations to characterize the current-flux characteristics. From our results it has been observed that, the persistent current exhibits ϕ_0/p periodicity in a p -fold twisted loop, and this periodicity of the current can be controlled nicely by tuning the total number of twist in the loop. Such a peculiar behavior is completely opposite to that of the conventional one-channel rings or multi-channel cylinders, where the persistent current always exhibits ϕ_0 flux-quantum periodicity. Our theoretical results in this article might be helpful to illuminate some of the unusual phenomena of persistent currents which have been observed in the twisted geometries.

Throughout our investigation, we have neglected the effect of electron-electron (e-e) correlation since the inclusion of the e-e correlation will not provide any new significant result in our present study.

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